though this engine does not represent by any means the current state of the art, its development has served to delineate those technical factors that affect engine performance and has resulted in a proven and reliable engine for the intended space test. These studies have served as an effective tool for the demonstration of the feasibility of cesium ion engines for space propulsion (at least as far as ground tests can so demonstrate) and have resulted in design parameters producing considerably improved engine performance. The authors believe, for example, that efficient, compact, cesium contact ion engines operating at an ionizer current density of 15 ma/cm^2 and incorporating improved ionizer heating techniques, propellant feed system, laboratory-proven neutralization systems, etc., can be made with confidence to exhibit performance values of less than 200 kw/lb in the 5000-sec specific impulse region and less than 250 kw/lb in the 10,000-sec specific impulse range.

Future work, which is essential to further improvement in engine performance in order to make such units available for space propulsion, includes studies of reliability over extended periods of time, space flight tests of certain critical engine characteristics, ionizer research, etc.

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Interaction Between Sound and Flow: Stability of T-Burners[1]

F. T. McClure,[2] R. W. Hart,[3] and R. H. Cantrell[4]
Johns Hopkins University, Silver Spring, Md.

The question of the effect of gas flow on the damping of an acoustic cavity with an orifice is of considerable importance in connection with the experimental determination of the acoustic admittance of burning solid propellants. The present investigation considers a center-vented rigid-walled cylindrical cavity that contains burning propellant at either or both ends. The only acoustic loss mechanism considered is that of the orifice. It is found that the presence of flow will, in general, have a significant effect on the conditions necessary for the acoustic stability of such a cavity and on the interpretation of the growth and decay rates in terms of the acoustic admittances characterizing the ends of the cavity.

ROCKET motors sometimes break into acoustic oscillation of such amplitude that the consequences are devastating to the performance and even to the integrity of the motor. This phenomenon has led a number of investigators to the study of the mechanics by which the energy of burning propellants is converted to high amplitude sound. In the considerable literature that has resulted from these studies, attention has been focused on the gains and losses presented by the various boundaries of the rocket cavity to the sound field. In general, the approach has been to attempt to represent these boundaries acoustically as admittances and then to discuss the problem of stability in the same manner as one would any ordinary acoustic system of this particular geometry.

More careful consideration, however, reminds one that the cavity is fundamentally somewhat different from those usually treated. It contains a mean flow field. This has consequences not usually encountered in describing resonances in acoustic cavities. One way of looking at the problem is to say that acoustic energy in such a cavity is propagated not only by the usual mechanical transfer but also by convective transport.

In assessing stability, one must account for the energy transferred by both of these mechanisms. For example, some regions of the cavity might have the ability to transform convectively transported energy. They would then, in a sense, appear to be "virtual amplifiers" of sound and would have to have admittances (which are intimately connected with mechanical transport) assigned to their surfaces accordingly. These assignments obviously would not be the same as would be made in the absence of the flow field. For example, insofar as the sound field is concerned, an orifice region could be an active, rather than a passive, acoustic element. Such behavior is implied by calculations of nozzle admittances that show negative real parts (1, 2).[5]

As an example of the modification that a flow field makes in stability criteria, consider the case of a propellant surface discharging gas into a cylinder cavity that is centrally vented to the outside (T-burner), as in Fig. 1. Attention will now be focused on the axial modes of this system.

If one neglects flow field effects, then, in the absence of any losses elsewhere in the system, stability would be determined

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[2] Chairman, Research Center, Applied Physics Laboratory.

[3] Supervisor, Theoretical Study Group, Research Center, Applied Physics Laboratory.

[4] Physicist, Theoretical Study Group, Research Center, Applied Physics Laboratory. Member AIAA.

[5] Numbers in parentheses indicate References at end of paper.

by the requirement that the propellant surface attenuate rather than amplify the mechanical wave. That is

$$Re\ Y_p > 0$$

where Y_p is the specific acoustic admittance of the propellant surface, and *Re* stands for real part.

It will be shown in this paper that the presence of the flow field modifies the criterion for this simple case to read

$$Re\left\{\frac{Y_p}{1 + \bar{\rho} v_p Y_p}\right\} > \frac{v_p}{\gamma \bar{P}}$$

where $\bar{P}$ and $\bar{\rho}$ are the mean pressure and density, γ is the specific heat ratio, and v_p is the mean speed at which the product gases leave the combustion region. Ordinarily one will expect Y_p to be small enough so that the second term in the denominator on the left will be negligible, so that the criterion will further simplify to

$$ReY_p > v_p/\gamma\bar{P}$$

This implies that the system can be unstable even if $Re\ Y_p > 0$, i.e., even though the propellant surface attenuates sound. This merely reflects the earlier point about the appearance of "virtual amplifiers" in flow systems.

Now the foregoing example is an over-simplified representation of the burning used by a number of workers to study the surface admittance of burning propellants, and therefore the correction for flow is not just an academic question. For example, the suggestion of Horton (3) that admittances could be obtained directly from the measurements of the growth rate of oscillation during burning and the decay rate after burning requires that one recognize these flow effects and properly account for them in the reduction of the data. It should be noted that the value of the propellant admittance and the flow correction terms given previously are of the same order.

Of course, the mean flow field introduces problems other than the one just stressed. For example, there may be rather severe gradients in temperature and flow fields at the metal walls of the cavity, and these "boundary layers" are not those usually encountered by the acoustician. Their properties as dampers (or perhaps even amplifiers) of sound have not been characterized. In this paper, the authors will not attempt to handle such problems but will confine themselves to those problems arising from the necessity of venting the product gases through an orifice.

Formulation of the Problem

The essence of the problem of determining the linear stability of the axial modes of a cavity such as that in Fig. 1 is that the cavity will present to the faces of the propellant a critical value for the real part of the specific admittance. If the real part of the propellant admittance is greater than this critical value, the cavity will be stable.

Of course, this critical admittance will contain, as indicated earlier, contributions arising from the mean flow field in the cavity. In general, these contributions will be most difficult to evaluate. However, if the flow field is everywhere well down in the subsonic regime, so that terms of the order of the square of the Mach number are negligible, certain simplifications occur which make the problem more tractable. Fortunately this situation does occur in many experimental burners. In cases where higher Mach numbers occur, the situation is much more complicated, as illustrated by the attempts to handle the acoustic properties of sonic nozzles (1, 2).

The treatment to be given here will be restricted to cases where the flow is everywhere in the low subsonic regime. Under these circumstances, the mean flow may be treated as incompressible. Viscosity, heat conduction, and inhomogeneities in the gas will be neglected also. It will also be assumed that the mean flow field has no curl.

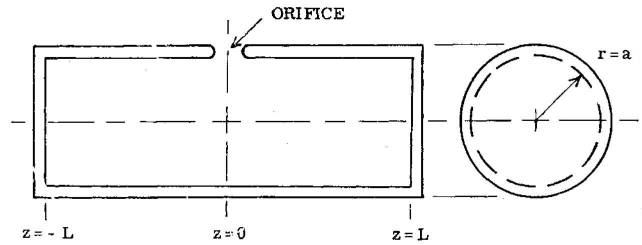

Fig. 1 Schematic drawing of centrally vented cavity (T-burner)

One further feature of the treatment should be mentioned. Although the authors might have gone on to the direct solution of the problem of the acoustic description of the situation, they have found it more convenient to use a technique that in a sense, is more powerful. It turns out that it is possible to transform the problem into the mathematical equivalent of a problem with no flow. This transformation is especially valuable since it gives the desired answers, using the ordinary acoustical concepts, without detailed reference to the physical interaction between the acoustic and flow fields and without requiring the introduction of any new mathematical methods. Although the treatment avoids these details, it also fails to illuminate them, and this may be unsatisfying to someone who wishes a dissection of the problem. Of course, each region of interest might be examined by the same (or another) method to provide such a dissection.

The method of attack is straightforward. First, the steady-state quantities such as pressure, velocity potential, and density will be perturbed about their steady-state values. Then the conservation equations are written in linear perturbation form. Two of these equations are then used to eliminate the pressure and density in order to obtain a partial differential equation in the velocity potential alone. It will then be noted that a suitable transformation of variables leads to a partial differential equation of the Helmholtz type such as describes ordinary no-flow acoustics. Finally, the boundary conditions also will be transformed, and it will become apparent that the problem with flow has been transformed into an analogous problem without flow.

The steps leading to the partial differential equation for a perturbed velocity potential are carried out in Ref. 4. The results in the present notation are (when terms of order M^2 are neglected)

$$\nabla^2\tilde{\varphi} - 2ik\mathbf{M}\cdot\nabla\tilde{\varphi} + k^2\tilde{\varphi} = 0 \quad [1]$$

and

$$\tilde{\epsilon} = -\{ik\gamma\tilde{\varphi} + \gamma\mathbf{M}\cdot\nabla\tilde{\varphi}\} \quad [2]$$

where $\mathbf{M}$ is the mean flow Mach number (vector), $k = \omega/c$, ω is the angular frequency, and c is the sound velocity. The acoustic pressure is $\epsilon\bar{P}$, and the acoustic velocity potential is $c\varphi$. Tilda quantities are Fourier amplitudes satisfying expressions of the form

$$\varphi = Re(\tilde{\varphi}e^{i\omega t}) \quad [3]$$

Transformation to the Helmholtz equation may be effected by defining a new variable $\Lambda(x, y, z)$, such that

$$\tilde{\varphi} \equiv \Lambda e^{(ik/c)\Psi} \quad [4]$$

where Ψ is the velocity potential for the mean flow. Then

$$\nabla\tilde{\varphi} = e^{(ik/c)\Psi}\{ik\Lambda\mathbf{M} + \nabla\Lambda\} \quad [5a]$$

and

$$\nabla^2\tilde{\varphi} = e^{(ik/c)\Psi}\{\nabla^2\Lambda + 2ik\mathbf{M}\cdot\nabla\Lambda\} \quad [5b]$$

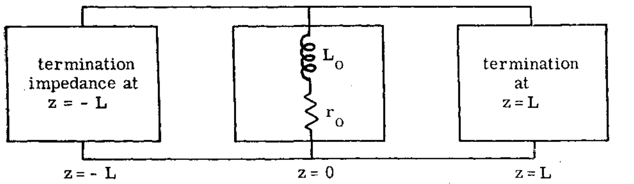

Fig. 2 Transmission line representation of a centrally vented cavity

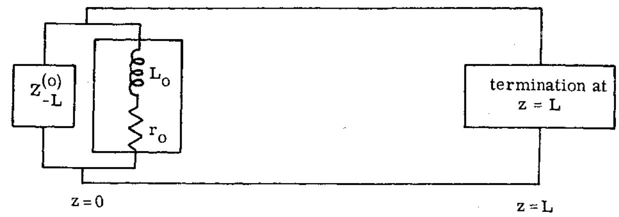

Fig. 3 Equivalent circuit for Fig. 2

Substitute Eqs. [4] and [5] into Eq. [1] to obtain

$$\nabla^2\Lambda + k^2(1 + 2M^2)\Lambda = 0$$

and since $2M^2$ may be neglected compared with unity, the partial differential equation for Λ is

$$\nabla^2\Lambda + k^2\Lambda = 0 \quad [6]$$

This equation is an ordinary Helmholtz equation for a stationary medium. The reader will thus note that the flow effects have been transferred to the boundary conditions for the transformed problem.

Under the conditions postulated, the vented cavity illustrated in Fig. 1 has three kinds of boundaries, namely, 1) rigid boundaries, where the component of velocity normal to the surface is zero, 2) burning boundaries, where the gas flow is normal to the surface and the acoustic properties are prescribed in terms of a specific acoustic admittance, and 3) boundaries at infinity,[6] where the solution must correspond to outgoing waves. This last condition is a specification that there are no sound sources exterior to the cavity. The transformation of each of these boundary conditions will be considered in turn.

For rigid walls, one has

$$\nabla\tilde{\varphi}\cdot\mathbf{1}_S = 0$$

where $\mathbf{1}_S$ is the unit surface vector (in the outward drawn normal direction). By using Eq. [5a] and noting that for a rigid surface $\mathbf{M}\cdot\mathbf{1}_S = 0$, one sees that

$$(\nabla\Lambda\cdot\mathbf{1}_S) = 0$$

at any rigid surface. Thus, rigid surfaces appear in the transformed problem also as rigid surfaces.

In the original problem, the boundary condition at a transpiring surface would have been stated in terms of the specific acoustic admittance of that surface, viz.,

$$Y = (c/\bar{\epsilon}\bar{P})\nabla\tilde{\varphi}\cdot\mathbf{1}_S \quad [7a]$$

In the transformed problem, state the boundary condition in terms of a transformed admittance Y' as

$$Y' = \frac{\nabla\Lambda\cdot\mathbf{1}_S}{-i\omega\bar{\rho}\Lambda} \quad [7b]$$

so that the value of Y' implied by the value of Y will now be sought. Using Eqs. [7a,7b, 4, 5a, and 2] and making use of the fact that $(\mathbf{M}\cdot\mathbf{1}_S) = -(v_p/c)$ on such surfaces (where v_p is the mean speed of the gas issuing from the burning zone), one obtains

$$Y' = \frac{Y}{1 + v_p\bar{\rho}Y} - \frac{v_p}{\gamma\bar{P}} \quad [7c]$$

It is clear from the definition of Λ that the condition that φ correspond to outgoing waves at infinity will pertain also to Λ.

The transformation of the flow problem into an analogous no-flow problem has now been completed. The criterion for the stability of the field then will be implied by the standard criterion for the stability of the Λ field.

[6] Note that the Helmholtz equation is being applied to the space both "inside" and "outside" the cavity, so that the exits are not boundaries. This, of course, implies that the outside space also has the propellant gas as its medium.

The Transformed Problem

Attention will now be directed to the solution of the transformed (no-flow) problem. Advantage will be taken of the fact that for axial modes the acoustic wavelengths will ordinarily be long compared with the diameter of the cavity and the orifice, and therefore the conventional lumped circuit approximations can be used to describe the orifice, except in certain cases to be noted.

Attention will be confined to low frequency modes, and the acoustic admittance at the plane $z = L$ will be asked for. Recall (5,6) that, if the orifice is small enough that the pressure across its entrance can be regarded as uniform, then the acoustic problem is analogous to the simple electrical transmission line problem of Fig. 2.

The orifice impedance corresponding to a "zero-length" hole corresponds to an inductance (5)

$$L_0 = C_L(8a_0\bar{\rho}/3\pi A_0)$$

(giving rise to a reactance $i\omega L_0$) and a radiation resistance

$$r_0 = C_r(\rho c/2A_0)(\omega a_0/c)^2$$

where a_0 is the radius of the hole, A_0 is the hole area, c is the sound velocity, ρ is the mean gas density, and C_L, C_r are coefficients that are unity for flanged holes and otherwise vary somewhat from unity, depending on the configuration exterior to the cavity.

At this point, it will be illuminating to examine the effect of the termination at $z = -L$ upon the acoustic properties of the right half of the chamber. For this purpose, the termination at $z = -L$ may be represented as an equivalent impedance $Z^{(0)}{}_{-L}$ at $z = 0$. Then the equivalent circuit for the right half of the cavity is that shown in Fig. 3.

It is quite clear that the magnitude of the loss through the orifice will be determined by the amount of current which flows through the radiation resistance r_0. Thus, if $Z^{(0)}{}_{-L}$ should be capacitive, and if the frequency should be such that resonance occurs, there would be a large loss because of the high circulating current in the resonant L-C circuit. On the other hand, if $Z_{-L}{}^{(0)}$ should be zero, the orifice impedance is short circuited. In such an event, no current flows through the orifice branch, and there is no acoustic loss.

This lossless condition corresponds to a pressure node at the orifice. If the left end of the cavity is rigid, for example, this means that, for a frequency such that the left cavity is one quarter of a wavelength long, there will be no loss of acoustic energy out of the orifice. This condition is a well-known one and is mentioned, for example, by Rayleigh (6). (Of course, the loss will not actually entirely vanish for this case; the lumped circuit description has become invalid, since there is now a pressure node located at the orifice, and the assumption of uniform pressure distribution over the orifice is invalidated.)

In general, there will be two cases of special interest, the first corresponding to one rigid end and one burning end, and the second corresponding to both ends terminated with the same burning propellant.

Case 1

Let the left end of the cavity be rigid. Then one requires the value of admittance at the right end which would correspond to neutral stability. The calculation is too elementary to be reproduced here, but the result is that the critical value of the specific acoustic admittance at $z = L$ is given by

$$\bar{\rho}cY_L' = \frac{2i\sin kL\cos kL + (\bar{\rho}c/AZ_0)\cos^2 kL}{\sin^2 kL - \cos^2 kL - i(\bar{\rho}c/AZ_0)\sin kL\cos kL} \quad [8]$$

where Z_0 is the "analogous" impedance $(i\omega L_0 + r_0)$ of the orifice, and A is the cross sectional area of the cavity. It should here be noted especially that Y_L' vanishes for frequencies such that kL is an odd multiple of $\pi/2$, corresponding to the fact that for these frequencies there would be a pressure node at the orifice. Of course, Eq. [8] becomes invalid at these frequencies, where the lumped circuit approximation fails because the pressure cannot be regarded as uniform over the orifice.

Case 2

Let the admittance of the two ends of the cavity be equal. Then the critical value of the specific acoustic admittance is readily found to be

$$\bar{\rho}cY_L' = \frac{-i\sin kL - (\bar{\rho}c/2AZ_0)\cos kL}{\cos kL + (i\bar{\rho}c/2AZ_0)\sin kL} \quad [9]$$

Eq. [9] is an incomplete solution, however, because it characterizes only modes that are symmetric in pressure, whereas it is quite obvious that, just as for Case 1, a possible solution to the cavity problem should correspond closely to modes for which (kL) is an odd multiple of $\pi/2$, because a pressure node would lie on the orifice at such frequencies.

In order to complete the solution by finding the admittance for pressure antisymmetric modes, one must modify the lumped circuit description. Instead of considering the pressure to be absolutely uniform in the region of pipe near the orifice, suppose that the pressure and velocity vary linearly in this region. The development is quite simple.

Just as for the usual lumped parameter method, one assumes that outside the orifice region $(-l \leqslant z \leqslant l')$ the field is axial and is given by

$$v = \frac{D_0}{\bar{\rho}c}\left\{e^{-ikz} - \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikz}\right\}$$

$$p = D_0\left\{e^{-ikz} + \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikz}\right\} \quad [10]$$

on the right-hand side of the cavity, $z \geqslant l'$ and

$$v = \frac{-D_1}{\bar{\rho}c}\left\{e^{ikz} - \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{-ikz}\right\}$$

$$p = D_1\left\{e^{ikz} + \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{-ikz}\right\} \quad [11]$$

on the left-hand side of the cavity, $z \leqslant -l'$, where p is the acoustic pressure, and D_0 and D_1 are constants to be determined by applying the continuity conditions across the orifice region. From the mass equation

$$\int \bar{\rho}\mathbf{v}\cdot dS = -\int \frac{\partial\rho}{\partial t}\,dV$$

(where dV is the volume element), and the approximation that the field quantities vary linearly in this region, one finds

$$v_{-l'} - v_{l'} - (A_0/A)v_0 \approx i\omega(\sigma_{l'} + \sigma_{-l'})l' \quad [12]$$

From the z component of the momentum equation

$$\int p d\mathbf{S} = -\int \frac{\partial}{\partial t}(\bar{\rho}\mathbf{v})dV \quad [13]$$

one finds

$$(-p_{-l'} + p_{l'}) \approx -i\omega\bar{\rho}(v_{l'} + v_{-l'})l'$$

From the energy equation, $\sigma = p/\gamma\bar{P}$. Substituting the expressions for v and p into the mass and momentum continuity expressions, one finds

$$-(D_0 + D_1)\left[e^{-ikl'} - \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikl'}\right] = (D_0 + D_1)\left[e^{-ikl'} + \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikl'}\right] \times \left[\frac{i\omega l'}{c} + \frac{\bar{\rho}c}{2Z_0A}\right] \quad [14]$$

$$(D_0 - D_1)\left[e^{-ikl'} + \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikl'}\right] = (D_0 - D_1)\left[e^{-ikl'} - \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikl'}\right]\left[-i\frac{\omega l'}{c}\right] \quad [15]$$

Thus, either $D_0 = D_1$ and

$$-\left[\frac{\bar{\rho}c}{2AZ_0} + \frac{i\omega l'}{c}\right] = \frac{e^{-ikl'} - [(1-\bar{\rho}cY_L')/(1+\bar{\rho}cY'_L)]e^{ikl'}e^{-2ikL}}{e^{-ikl'} + [(1-\bar{\rho}cY_L')/(1+\bar{\rho}cY'_L)]e^{ikl'}e^{-2ikL}} \quad [16]$$

which corresponds to the previously obtained result as $kl' \to 0$, or $D_0 = -D_1$ and

$$e^{-ikl'} + \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikl'} = -\left(\frac{i\omega l'}{c}\right)\left[e^{-ikl'} - \left(\frac{1-\bar{\rho}cY'_L}{1+\bar{\rho}cY'_L}\right)e^{-2ikL}e^{ikl'}\right] \quad [17]$$

which corresponds to the desired solution for modes having an antisymmetric pressure distribution. Since the dimension of the orifice is ordinarily very small compared with the diameter of the cavity $[(kl') \ll 1$, so that one neglects terms of order $(kl')^2]$, one obtains

$$\bar{\rho}cY'_L = i\cot(kL) \quad [18]$$

which is just the result that would be obtained if there were no orifice in the cavity. Thus, to this approximation, the existence of a small central orifice has no effect and introduces no loss for antisymmetric pressure modes. A more sophisticated, variational analysis that will be published in another connection elsewhere allows an estimate of the orifice losses for the antisymmetric pressure case. In that analysis, for an orifice that is an annular gap of length l, the essence of the result is that

$$Re\ Y'_L \simeq -\frac{(2m+1)^2\pi^4 l^4}{256\,(\ln 2)^2\bar{\rho}c\ a^2L^2} \quad [19]$$

where $m = 0, 1, 2, \ldots$

Considering a typical geometry with $a/l = 10$, $L/a = 10$, and evaluating Eq. [19] for the lowest mode $(m = 0)$, one finds

$$Re\ Y'_L \simeq -(10^{-6}/\bar{\rho}c)$$

which is about three orders of magnitude less than the propellant specific admittance normally expected. One sees that for the lower axial modes of this symmetry the orifice losses would be negligible. Thus the more sophisticated treatment does not lead to results significantly different from the modified lumped circuit analysis for the lower axial modes. For the higher modes, of course, Eq. [19] would predict significant losses.

Discussion

The treatment given in the preceding section allows a detailed analysis of the structure of the axial modes to be expected (with no wall or volume losses) for a cavity such as that shown in Fig. 1. The authors will not go into that detail here but will merely summarize the results. There turn out to be three classes of axial modes. First, there are the pressure symmetric modes with a pressure antinode at the orifice. These are highly damped and will not be expected to be excited except under very unusual conditions. Second, there are the pressure antisymmetric modes in which a pressure node occurs at the orifice. The lower members of this family are only very slightly damped by the orifice, and these modes will be the ones expected under driving conditions. Third, there is a set of pressure-symmetric modes that have a very low pressure at the orifice, with a cusplike distribution in its vicinity. The lower members of this family are only moderately damped, however; they will not commonly be observed under driving conditions because of the predominant preference for the antisymmetric mode. They may, however, appear during the decay of oscillations.

Turning now to the lower pressure antisymmetric modes that are to be expected under driving conditions, note that the orifice loss (for subsonic orifices) will usually be small, and, therefore, in the absence of other loss, the criterion for instability will be

$$Re\ Y_p' < 0$$

But by Eq. [7c], this becomes

$$Re\left\{\frac{Y_p}{1 + \bar{\rho}v_p Y_p} - \frac{v_p}{\gamma \bar{P}}\right\} < 0$$

Except in the vicinity of resonant motion in the solid propellant, Y_p will be sufficiently small so that, to a good approximation, this criterion may be written as

$$Re\{Y_p - (v_p/\gamma\bar{P})\} < 0 \qquad [20]$$

This means that the system can be unstable even for positive values of $Re\ Y_p$, and in fact the results are such that instability down to quite low frequencies is to be expected for such modes of T-burners except for propellants with a very low pressure dependence of their burning rates. At low frequencies $Re\ Y_p \to (v_p/\bar{P})(1 - n)$, where n is the pressure exponent in the burning rate law (7). Then, for these idealized conditions the criterion (Eq. [20]) for instability becomes

$$n > (\gamma - 1)/\gamma \simeq 0.2$$

A point of especial significance for the determination of propellant specific admittances by the growth rate of oscillations in T-burners is that the appropriate instability criterion is Eq. [20] rather than $Re\ Y_p < 0$. This means that the numbers obtained from such experiments, not correcting for flow, really represent the left-hand side of Eq. [20] more nearly than $Re\ Y_p$.

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See December 1962 ARS Journal, page 1845, for further details